

HOW TO DIVERSIFY THE TAX-SHELTERED EQUITY FUND

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ABSTRACT

Equity mutual funds generally put much emphasis on growth stocks as opposed to income stocks regardless of whether funds are tax-sheltered or unsheltered. This paper develops a portfolio model for a tax-sheltered fund incorporating the difference between growth and income stocks in the presence of differential effective taxes on dividends and capital gains. The results show that a superior performance can be obtained for a tax-sheltered fund by creating a segregated portfolio with greater emphasis on income stocks.

I. INTRODUCTION

Aggregate data of stock portfolios held by mutual funds reveal great emphasis on growth stocks as opposed to income stocks in both tax-sheltered and unsheltered funds. This phenomenon has persisted through the eighties in the face of a dramatic growth in the combined size of those portfolios and a major tax reform. According to data published by the Investment Company Institute (*Mutual Fund*

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Fact Book, 1983–1989) and reported in Table 1, of the entire tax-sheltered stock portfolio held at the end of 1988, 51.8% was invested in funds specializing in growth stocks, 36.2% in funds holding both growth and income stocks, and 12.0% in funds emphasizing income stocks. The respective proportions for unsheltered funds were similar: 46.2, 43.8, and 10.0%. These figures are only slightly different from those reported for the end of 1985 or 1982. The similarity in the portfolio composition of sheltered and unsheltered equity funds is not surprising in view of the common practice of mutual funds not to segregate sheltered portfolios, but to identify sheltered and unsheltered claims on the same portfolios. This paper questions the consistency of this policy with the objective of maximizing tax-sheltered return. If the market is dominated by tax-paying investors, as evidence suggests it is, risk-adjusted rates of return will approach uniformity after tax, but not before tax. To the extent that capital gains are taxed at preferential effective rates, as evidence suggests they are even under the 1986 Tax Reform Act, uniform posttax returns imply higher risk-adjusted pretax returns of those stocks in which a greater proportion of the return is paid in dividends.

The extent of market domination by tax-paying investors is a difficult empirical issue. Peterson, Peterson, and Ang (1985) estimate at 40% the average marginal tax rate on dividends received by U.S. taxpayers in 1979. This finding is consistent with the figures reported in Table 1, showing that the total stock portfolio held by tax-sheltered funds has been small relative to that held by unsheltered ones. Mutual funds are major players among tax-sheltered institutional investors. Direct evidence of a substantial spread between the tax rates of dividends and capital gains is offered by Kalay (1982) based on ex-dividend share price behavior and stock ownership clientele. In a much-cited study, Miller and Scholes (1978) hypothesize that arbitrage by shareholders allows substitution of capital gains tax for the dividend tax, generating tax saving by exposure to a

Table 1. Mutual Equity Funds by Tax Treatment and Investment Objective: Year-End Assets^a

<i>Investment objective</i>	<i>Tax Treatment</i>		<i>1985</i>		<i>1988</i>	
	<i>Sheltered</i>	<i>Unsheltered</i>	<i>Sheltered</i>	<i>Unsheltered</i>	<i>Sheltered</i>	<i>Unsheltered</i>
Billions of dollars	5.1	51.3	17.9	93.3	38.8	129.6
Percentages						
Growth ^b	62.4	49.4	61.7	47.3	51.8	46.2
Growth & income	26.3	40.3	33.4	41.8	36.2	43.8
Income	11.3	10.3	4.9	10.9	12.0	10.0

Notes: ^aThe reported figures exclude about 10% of equity funds not classified according to these investment objectives.

^bThis category includes "aggressive growth" funds.

lower tax rate. The effective tax rate is then further reduced by deferred realization of capital gains. This hypothesis is inconsistent with the evidence cited above, and also at odds with additional evidence that the proposed arbitrage is empirically insignificant. In particular, the law creating the loophole described by Miller and Scholes has been in effect only since 1969 and, as demonstrated empirically by Feenberg (1981), even since that time could not have affected more than 0.1% of all taxpayers receiving dividends, or 3% of personal dividend income.

The possibility of a substantial spread between the effective dominant tax rates of dividends and capital gains persists under the 1986 Tax Reform Act to the extent that taxable shareholders, whether individuals or institutions, lower the effective capital gains tax rate by deferred realization of gains. Judging by the history of the capital gains tax, that spread is likely to increase again by legislation lowering the statutory rate of this tax relative to that of dividends (see Katz, 1981).

A previous study by Yaari and Fabozzi (1985) investigates the theoretical relationship between the growth-versus-income composition of return and the total rate of return of a tax-sheltered fund. Focusing on a single stock, the authors show that an inverse relationship exists between the stock's per-share growth rate and the fund's untaxed return. In abstracting from risk and effects of diversification, that study offers merely a conjecture that the composition of return should play a role in selecting the portfolio held by the tax-sheltered fund. This paper follows up by formally incorporating the effect of growth versus income on the *optimal portfolio* of such a fund.

The theoretical foundation for the proposed method of diversification is laid in the works of Brennan (1970) and Elton and Gruber (1978). Brennan extends the capital-asset-pricing model (CAPM) to an environment of personal taxes, but does not investigate the implications for individual investors in diverse tax brackets. His treatment offers no guidance to the sheltered fund whose zero marginal tax rates are substantially below those which according to recent evidence dominate the stock market. Elton and Gruber derive the optimal portfolio of an investor belonging in a tax bracket that is different from that dominating the market, without specific reference to the CAPM. Their treatment is impractical for large investors such as mutual funds holding highly diversified portfolios. The results presented in this paper are useful to such investors in a number of ways. First, they provide a correct and practical method for constructing an optimal stock portfolio tailored for tax-sheltered funds. The portfolio formulated with consideration of preferential treatment of capital gains via reduced tax rates or opportunities for tax deferral may substantially differ in composition and performance from one that is selected without reference to the effect of personal taxes or the unique tax position of sheltered funds. Second, a portfolio selected by the proposed method can serve as a benchmark for evaluating the performance of such funds. Third, with preferential treatment of capital

gains, our results show the potential benefits from segregating tax-sheltered funds in specialized portfolios, in lieu of the common practice of combining sheltered and unsheltered funds in the same portfolios. The latter practice is consistent with the aggregate figures reported in Table I, suggesting that shareholders of tax-sheltered funds were ill-served under the old tax law, and perhaps under the new law as well.

II. THE MODEL

To set the stage for the analysis of diversification by tax-sheltered funds, we first present a synthesis of the posttax CAPM and the Elton–Gruber analysis of portfolio composition.

The assumptions required for the standard tax-free CAPM are supplemented by the following assumptions required for the posttax version of that model:

1. Investors have homogeneous expectations on the pretax rate of return and dividend yield of any security j denoted by R_j and δ_j , respectively; however, expectations on the posttax return are heterogeneous due to different tax rates paid by individuals.
2. Dividends are fully anticipated, so that the uncertainty of return is attributed entirely to uncertain capital gains.

As seen by investor i , the aftertax expected return on security j is

$$\bar{R}_{ij}^{\wedge} = \delta_{ij} (1 - t_{di}) + \gamma_j (1 - t_{gi}) \quad (1)$$

where γ_j is the anticipated nonstochastic capital gain or one-period growth rate, and t_{di} and t_{gi} the effective personal tax rates on dividends and capital gains, respectively. Based on the pretax relationship (note that the pretax $\bar{R}_{ij} = \bar{R}_j$ is the same for all i)

$$\gamma_j = \bar{R}_j - \delta_j$$

the posttax return in Equation (1) is conveniently written as

$$\bar{R}_{ij}^{\wedge} = \bar{R}_j (1 - t_{gi}) - \delta_j (t_{di} - t_{gi}) \quad (2)$$

The optimization problem facing investor i is to identify the portfolio that would maximize the posttax excess return per unit risk

$$\text{Max: } \theta_i = (\bar{R}_{ip}^{\wedge} - R_{if}^{\wedge}) / \sigma_{ip}^{\wedge} \quad (3)$$

subject to the constraint

$$\sum_j X_{ij} = 1$$

An explicit statement of the objective function is obtained by spelling out the expected value of return on a portfolio

$$\bar{R}_{ip}^A = \sum_j X_{ij} [\bar{R}_j (1 - t_{gi}) - \delta_j (t_{di} - t_{gi})]$$

the standard deviation of return on a portfolio

$$\sigma_{ip}^A = \left[\sum_j X_{ij}^2 (1 - t_{gi})^2 \sigma_j^2 + \sum_j \sum_{\substack{k \\ k \neq j}} X_{ij} X_{ik} \sigma_{jk} (1 - t_{gi})^2 \right]^{1/2}$$

and the posttax opportunity return on the risk-free asset

$$R_{if}^A = R_f (1 - t_{di})$$

The absence of any constraint on the proportion of wealth X_{ij} invested by i in any risky security j reflects the absence of a restriction on short sales.²

Using the vector definitions

$$R = \begin{bmatrix} R_1 - R_f \\ \vdots \\ R_j - R_f \\ \vdots \\ R_n - R_f \end{bmatrix}, \quad D = \begin{bmatrix} \delta_1 - R_f \\ \vdots \\ \delta_j - R_f \\ \vdots \\ \delta_n - R_f \end{bmatrix}, \quad X_i = \begin{bmatrix} X_{i1} \\ \vdots \\ X_{ij} \\ \vdots \\ X_{in} \end{bmatrix}$$

and denoted by V the portfolio variance-covariance matrix,³ we obtain the following vector as a first-order solution to Equation (3), representing the demand equation of investor i for a risky portfolio:

$$Z_i = V^{-1} (R - Dt_i) \quad (4)$$

where: $Z_i = \lambda_i X_i$
 $\lambda_i = (X_i' R - X_i' D t_i) / X_i' V X_i$
 $t_i = (t_{di} - t_{gi}) / (1 - t_{gi})$

The market clearing condition is set by

$$\sum_i W_i X_i / \sum_i W_i = X_m \quad (5)$$

where X_m is the value-share column vector of the market portfolio and W_i the wealth invested in the risky portfolio by investor i . Summing up individual demand equations over all i and rearranging the result, we obtain the following equilibrium vector of security *excess returns*:

$$R = HVX_m + t_m D \quad (6)$$

containing the scalar

$$H = \sum_i W_i / \sum_i (W_i / \lambda_i)$$

and the average market tax rate

$$t_m = \sum_i (t_i W_i / \lambda_i) / \sum_i (W_i / \lambda_i)$$

Substitution of Equation (6) in Equation (4) generates the optimal portfolio of risky assets demanded by investor j under conditions of market equilibrium:

$$Z_j = HX_m + V_j^{-1} D(t_m - t_j). \quad (7)$$

III. THE EFFECT OF GROWTH

The optimal share of security j in the individual's risky portfolio may be smaller or greater than the share of that security in the market portfolio, depending on the value of the second term in Equation (7):

$$Z_{ij} \gtrless HX_{mj} \quad \text{according to} \quad V_j^{-1} D(t_m - t_j) \gtrless 0 \quad (8)$$

In words, the relationship between the private and market shares of a security depends on the relationship between the private tax rate t_j and the tax rate dominating the market equilibrium, t_m . Equality of these shares is obtained in a taxless environment or if private and market rates are equal, $t_j = t_m$. (This condition may be met in unsheltered mutual funds.)⁴

We are interested in the special case of tax-sheltered funds, where $t_i = 0$ implies $t_m - t_i > 0$, so that

$$Z_{ij} \underset{<}{\underset{>}{\geq}} HX_{mj} \quad \text{according to} \quad V_j^{-1}D \underset{<}{\underset{>}{\geq}} 0 \quad (9)$$

This shows that, relative to the market, the optimal share of security j in the sheltered portfolio depends on the average difference between the risk-free rate and the dividend yields of all securities.

Without further assumptions, the last result is too general for application. Under the limiting assumptions of a single-index CAPM, Equation (7) as applied to security j becomes

$$Z_j = HX_{mj} + C_j(t_m - t_i) \quad (10)$$

subject to

$$C_j = \frac{\beta_j}{\sigma_{ej}^2} \left(\frac{\delta_j - R_f}{\beta_j} - a \right)$$

where σ_{ej}^2 is the residual variance of security j measured in reference to the market index, β_j the security beta, and a the weighted average of $(\delta_j - R_f)/\beta_j$ across all securities—a value taken by investor i as a parameter. Under $t_i = 0$, the condition expressed in Equation (9) is simplified to

$$Z_{ij} \underset{<}{\underset{>}{\geq}} HX_{mj} \quad \text{according to} \quad C_j \underset{<}{\underset{>}{\geq}} 0$$

showing that the relative share of security j in the sheltered portfolio is affected by its relative dividend yield as well as its relative risk.

Using a deterministic valuation model, Yaari and Fabozzi (1985) demonstrated a negative effect of stock growth on tax-sheltered return. Extrapolating this result to an uncertain environment, they claim without proof that sheltered funds may adversely affect their shareholders by concentrating in growth stocks. This conjecture is confirmed by partially differentiating Equation (10) with respect to δ_j :

$$\frac{\partial Z_{ij}}{\partial \delta_j} = \frac{t_m - t_i}{\sigma_{ej}^2} > 0 \quad (\text{subject to } t_i = 0) \quad (11)$$

Holding risk constant, a decrease in a stock's growth rate (an increase in its dividend yield) increases its optimal share in a sheltered portfolio.

IV. CONCLUSION

In view of the evidence that tax-sheltered funds are heavily invested in growth stocks in a market dominated by preferential tax treatment of capital gains, Equation (11) suggests that their performance can be improved by creating funds in segregated portfolios with greater emphasis on income stocks. The desirable proportions of individual stocks depend on the statistical model used. Assuming the validity of the single-index CAPM, the optimal proportions are calculated by Equation (10). Unlike stock diversification in a tax-free environment or one in which capital gains and dividends are subject to the same effective tax rates, investment policy based on Equation (10) requires data on stock growth rates (or payment ratios) and an estimate of the tax bracket dominating the market. Although available estimates of these parameters are likely to admit error to the analysis, a greater error is committed by overlooking the consequences of tax shelter, or ignoring altogether the effects of personal taxes.

NOTES

1. Allen (1982), Constantinides (1983,1984), and Stiglitz (1983) show that by adopting appropriate portfolio strategies, investors in a perfect market would avoid paying capital gains tax. However, Poterba (1986) demonstrates empirically that in the real market a large majority of the investing public do not engage in such tax-minimizing portfolio transactions, lowering the effective rate of realization of gains.

2. There is no legal restriction on short sales of securities by tax-sheltered mutual funds, and such transactions are common in funds labeled "super growth." Self-imposed restrictions on short sales would result in suboptimal diversification. The theoretical and computational implications of such a restriction are analyzed in Elton and Gruber (1984).

3. For the construction of this matrix in a world of personal taxes, see Brennan (1970) or Elton and Gruber (1984).

4. The standard posttax CAPM is obtained by applying the condition of Equation (6) to the market portfolio

$$R_m - R_f = H\sigma_m^2 + t_m(\delta_m - R_f)$$

solving for H and substituting the result back in Equation (6), we obtain in scalar notation

$$R_j = R_f + \beta_j[(R_m - R_f) - t_m(\delta_m - R_f)] + t_m(\delta_j - R_f)$$

where

$$\beta_j = \text{cov}(R_j, R_m) / \sigma_m^2.$$

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